March 22, 2016

M001: Mathematics

Homework 1

Problem 1
Let

$$\mathbf{A} = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 0 & 1 \\ 3 & 2 \\ -2 & 0 \end{bmatrix} \qquad \mathbf{a} = \begin{bmatrix} 1 & -2 & 0 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$$

Calculate the following expressions or give reasons why they are undefined:

$$\mathbf{AB},\ \mathbf{AB}^T,\ \mathbf{BA},\ \mathbf{B}^T\mathbf{A},\ \mathbf{Aa},\ \mathbf{Aa}^T,\ \left(\mathbf{Ab}\right)^T,\ \mathbf{b}^T\mathbf{A}^T$$

Problem 2
Solve the following system of linear equations.

$$\begin{bmatrix} 4 & 7 & 3 & 1 \\ 3 & 6 & 4 & 2 \\ 1 & -5 & 3 & 1 \\ -2 & 3 & 3 & -1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 31 \\ 35 \\ 4 \\ 9 \end{bmatrix}$$

Problem 3
For what values of a does the system of linear equations

$$\begin{bmatrix} 1 & 1 & 1 \\ -a & 1 & 2 \\ -2 & 2 & a \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

(a) have a unique solution, (b) infinitely many solution, (c) no solution. Hint: Start by calculating the determinant of the coefficient matrix.

Problem 4
Show that the matrix

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 3 & 6 & 9 \end{bmatrix}$$

cannot be inverted.

Problem 5
Find the inverse to the matrix

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

Problem 6
Consider a triangle with corner points $\mathbf{x} = [-1\ \ 2\ \ 5]$ $\mathbf{y} = [-3\ \ 2\ \ 1]$. Using a transformation matrix

$$\mathbf{T} = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}$$

with $\varphi = 60^\circ$, calculate by hand the corner points x^* and $\mathbf{y}^*$ in the rotated position (counterclockwise rotation).

$$\begin{bmatrix} x_1^* & x_2^* & x_3^* \\ y_1^* & y_2^* & y_3^* \end{bmatrix} = \mathbf{T} \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix}$$

Draw both triangles.

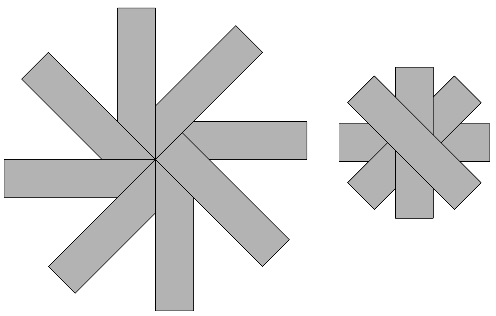
Problem 7

Use SCILAB or any other program of your choice to plot a 4 (width) x 1 (height) rectangle. Do this by generating a vector (column) of x -and y - values of the four corner points of the rectangle. Then use the transformation

$$\begin{bmatrix} x_1^* & x_2^* & x_3^* & x_4^* \\ y_1^* & y_2^* & y_3^* & y_4^* \end{bmatrix} = \mathbf{T} \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{bmatrix} \qquad \text{with } \mathbf{T} = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}$$

to rotate the rectangle about the bottom-left corner point (the point with $x = y = 0$). Select angles φ of 45, 90, 135, 180, 225, 270 and 315 degrees. Plot the rectangle in its rotated position. Repeat the problem by rotating the rectangle about its center.

If all goes well, the resulting figures should look like this

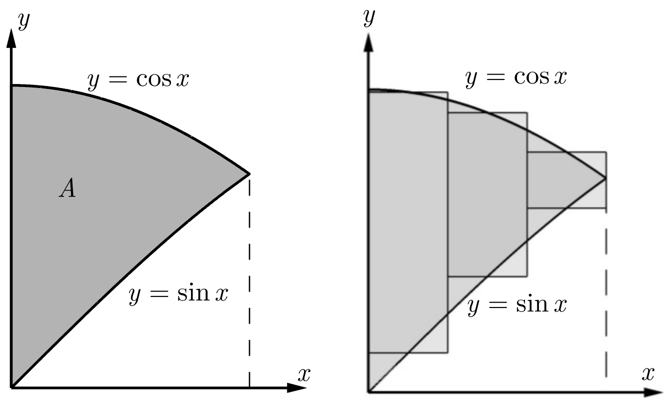


Problem 8

(a) Use SCILAB to find the x - and y -coordinates of the centroid of area A by subdividing the area into N rectangels (shown for $N = 3$). Test your algorithm using $N = 10$.

(b) Find the centroid by hand using integration.

Solution:



(a) $x_S = 0.26854$ $\qquad$ $y_S = 0.60402$

(b) $x_S = \dfrac{\frac{\pi}{4}\sqrt{2} - 1}{\sqrt{2} - 1} = 0.26730$ $\qquad$ $y_S = \dfrac{1}{4\left(\sqrt{2} - 1\right)} = 0.60355$

Problem 9

Plot the function

$$y_N(x) = \frac{4}{3}\pi + \sum_{n=1}^{N} \left(\frac{4}{n^2}\cos nx - \frac{4\pi}{n}\sin nx \right)$$

for $N = 5$ and $N = 10$ and $0 \le x \le 6$ (two curves in one plot; x -values spaced at 0.02, i.e.. 301 values for x).